ERRATA AND SIMPLIFICATION FOR HOCHBAUM AND RAO OR 2019

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Abstract. The purpose of this write-up is to correct an error in a lower bound used in [1], and to show that the corrections required do not affect the results. Another part of this write-up is a simplification and streamlining of the Fully Polynomial Time Approximation Scheme result in the paper.

1. Correcting an error. We recently found an error in the proof of Theorem 6 in [1]. This part of the write-up specifies the modifications required to address this error. The error is that the lower bound of the optimal value V^* was written incorrectly as $\frac{k+1}{2}\sum_{i=1}^n s_i = \frac{k(k+1)}{2}D$ where it should have been $\frac{k+1}{2k}\sum_{i=1}^n s_i = \frac{k+1}{2}D$. This error affects Algorithm 3, which is an ϵ-approximation algorithm. In order to correct it we change the scaling factor in Step 1 of Algorithm 3 from $\epsilon^2 D$ to $\frac{\epsilon^2 D}{k}$. The running time of Algorithm 3 with this adjusted scaling factor is still $O(\frac{n}{\epsilon^{2k}})$ for constant k. Hence, the results of the paper do not change. As an aside, we note that this running time is in fact fixed-parameter tractable for the parameter k.

We now list the changes in lemmas and formulas in Section 4.2 of [1] needed as a result of the modification of the scaling factor:

1. The right hand side of the ϵ-relaxed cascading constraints in (ϵ-relaxed RSP) should be changed from $\ell D + \epsilon k D$ to $\ell D + \epsilon D$ for $\ell = 1, ..., k$.
2. In Lemma 6, the bound should be changed from $V_k(\mathbf{x}) + \epsilon k D$ to $V_k(\mathbf{x}) + \epsilon D$.
3. In Theorem 2, the inequalities should be changed from

$$\epsilon^2 D g'(\mathbf{x}^{\mathrm{L}}) - \frac{k^2(k+1)D}{2}\cdot\frac{\epsilon}{1-\epsilon} \;\le\; g(\mathbf{x}^{\mathrm{L}}) \;\le\; \epsilon^2 D g'(\mathbf{x}^{\mathrm{L}}) + \frac{k(k+1)(k+2)D}{6}\cdot\frac{\epsilon}{1-\epsilon}$$

to

$$\frac{\epsilon^2 D}{k} g'(\mathbf{x}^{\mathrm{L}}) - \frac{k(k+1)D}{2}\cdot\frac{\epsilon}{1-\epsilon} \;\le\; g(\mathbf{x}^{\mathrm{L}}) \;\le\; \frac{\epsilon^2 D}{k} g'(\mathbf{x}^{\mathrm{L}}) + \frac{(k+1)(k+2)D}{6}\cdot\frac{\epsilon}{1-\epsilon}.$$

4. The value of $\delta(\epsilon)$, which appears in Theorem 3, Corollary 1, Theorem 5 and Theorem 6, should be changed from $\delta(\epsilon) = \frac{k(k+1)(2k+1)D}{3}\cdot\frac{\epsilon}{1-\epsilon}$ to $\delta(\epsilon) = \frac{(k+1)(2k+1)D}{3}\cdot\frac{\epsilon}{1-\epsilon}$.

Finally, with the updated value $\delta(\epsilon)$ and Lemma 6, in the proof of Theorem 6 the inequality $V(\hat{\mathbf{x}}) \le V^* + \frac{\delta(\epsilon)}{k} + \epsilon k D$ should be changed to $V(\hat{\mathbf{x}}) \le V^* + \frac{\delta(\epsilon)}{k} + \epsilon D$. Observe that the new $\delta(\epsilon)$ is $1/k$ times the original value, and the new second additional term ϵD is $1/k$ times the original one, $\epsilon k D$. Hence, using the corrected lower bound of V^*, which is also $1/k$ times the one that was used, we get the same bound for the ratio $V(\hat{\mathbf{x}})/V^*$. Therefore, Theorem 6 holds with the correction and the modified scaling factor.

2. A simplification for proving the approximation bound. We present here a streamlined version of Theorem 2, resulting in simplified inequalities and formulas in several lemmas and theorems.

We provide next the new version of Theorem 2 and its proof.

THEOREM 2.1. *For any assignment of large items $\mathbf{x}^L$ feasible for (scaled-modified-k-RSP$_1$), the values of the objective function with original and scaled sizes, $g(\mathbf{x}^L)$ and $g'(\mathbf{x}^L)$ respectively, satisfy,*

$$\frac{\epsilon^2 D}{k}\cdot g'(\mathbf{x}^L) - \epsilon k^2 D \;\le\; g(\mathbf{x}^L) \;\le\; \frac{\epsilon^2 D}{k}\cdot g'(\mathbf{x}^L) + \epsilon k^2 D.$$

Proof. Let $T = \frac{\epsilon^2 D}{k}$ denote the scaling factor. Recall that $s'_i = \lfloor \frac{s_i}{T} \rfloor$, so $T s'_i \le s_i < T(s'_i + 1)$.

So for any integer time j,

$$Q_j(\mathbf{x}^{\mathrm{L}}) = \sum_{i=1}^{n_{\mathrm{L}}} s_i x_{ij}^{\mathrm{L}} < T\cdot\sum_{i=1}^{n_{\mathrm{L}}}(s'_i+1)x_{ij}^{\mathrm{L}} = T\cdot\left(Q'_j(\mathbf{x}^{\mathrm{L}}) + \sum_{i=1}^{n_{\mathrm{L}}} x_{ij}^{\mathrm{L}}\right)$$

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The second term in the parentheses, $\sum_{i=1}^{n_{\mathrm{L}}} x_{ij}^{\mathrm{L}}$, must be less tan or equal to the number of large items, which is bounded by $\frac{k}{\epsilon}$. Therefore we derive the following inequality

$$Q_j(\mathbf{x}^{\mathrm{L}}) < T \cdot \left(Q'_j(\mathbf{x}^{\mathrm{L}}) + \frac{k}{\epsilon} \right) = T \cdot Q'_j(\mathbf{x}^{\mathrm{L}}) + \epsilon D \quad \text{for } j = 1, ..., k. \tag{2.1}$$

Using $s_i \geq T s'_i$ for any i, we get

$$Q_j(\mathbf{x}^{\mathrm{L}}) = \sum_{i=1}^{n_{\mathrm{L}}} s_i x_{ij}^{\mathrm{L}} \geq T \cdot \sum_{i=1}^{n_{\mathrm{L}}} s'_i x_{ij}^{\mathrm{L}} = T \cdot Q'_j(\mathbf{x}^{\mathrm{L}}) \quad \text{for } j = 1, ..., k. \tag{2.2}$$

Recall that the adjusted remainder of time τ is $\bar{R}_\tau(\mathbf{x}^{\mathrm{L}}) = \min_{\ell \geq \tau} R_\ell = \min_{\ell \geq \tau} \left(\ell D - \sum_{j=1}^{\ell} Q_j(\mathbf{x}^{\mathrm{L}}) \right)$, and that the scaled adjusted remainder of time τ is $\bar{R}'_\tau(\mathbf{x}^{\mathrm{L}}) = \min_{\ell \geq \tau} \left(\ell D' - \sum_{j=1}^{\ell} Q'_j(\mathbf{x}^{\mathrm{L}}) \right)$. We derive from inequality (2.1) that for any time τ,

$$\begin{aligned} \bar{R}_\tau(\mathbf{x}^{\mathrm{L}}) &= \min_{\ell \geq \tau} \left(\ell D - \sum_{j=1}^{\ell} Q_j(\mathbf{x}^{\mathrm{L}}) \right) \\ &\geq \min_{\ell \geq \tau} \left[\ell D - \sum_{j=1}^{\ell} \left(T \cdot Q'_j(\mathbf{x}^{\mathrm{L}}) + \epsilon D \right) \right] \\ &\geq \min_{\ell \geq \tau} \left[\ell D - T \cdot \sum_{j=1}^{\ell} Q'_j(\mathbf{x}^{\mathrm{L}}) \right] - \epsilon k D \\ &= T \cdot \bar{R}'_\tau(\mathbf{x}^{\mathrm{L}}) - \epsilon k D. \end{aligned} \tag{2.3}$$

And we derive from inequality (2.2) that for any time τ,

$$\bar{R}_\tau(\mathbf{x}^{\mathrm{L}}) = \min_{\ell \geq \tau} \left(\ell D - \sum_{j=1}^{\ell} Q_j(\mathbf{x}^{\mathrm{L}}) \right) \leq \min_{\ell \geq \tau} \left(\ell D - T \cdot \sum_{j=1}^{\ell} Q'_j(\mathbf{x}^{\mathrm{L}}) \right) = T \cdot \bar{R}'_\tau(\mathbf{x}^{\mathrm{L}}) \tag{2.4}$$

Using the inequalities (2.1) and (2.4), we prove the upper bound on $g(\mathbf{x}^{\mathrm{L}})$ as follows:

$$\begin{aligned} g(\mathbf{x}^{\mathrm{L}}) &= \sum_{j=1}^{k} (k - j + 1) Q_j(\mathbf{x}^{\mathrm{L}}) + \sum_{\tau=1}^{k} \bar{R}_\tau(\mathbf{x}^{\mathrm{L}}) \\ &< \sum_{j=1}^{k} (k - j + 1) \left(T \cdot Q'_j(\mathbf{x}^{\mathrm{L}}) + \epsilon D \right) + \sum_{\tau=1}^{k} T \cdot \bar{R}'_\tau(\mathbf{x}^{\mathrm{L}}) \\ &= T \cdot \left[\sum_{j=1}^{k} (k - j + 1) Q'_j(\mathbf{x}^{\mathrm{L}}) + \sum_{\tau=1}^{k} \bar{R}'_\tau(\mathbf{x}^{\mathrm{L}}) \right] + \epsilon D \cdot \sum_{j=1}^{k} (k - j + 1) \\ &\leq T \cdot g'(\mathbf{x}^{\mathrm{L}}) + \epsilon k^2 D. \end{aligned}$$

The lower bound on $g(\mathbf{x}^{\mathrm{L}})$ follows from inequalities (2.2) and (2.3):

$$\begin{aligned}
g(\mathbf{x}^{\mathrm{L}}) &= \sum_{j=1}^{k}(k-j+1)Q_j(\mathbf{x}^{\mathrm{L}}) + \sum_{\tau=1}^{k}\bar{R}_\tau(\mathbf{x}^{\mathrm{L}}) \\
&\geq \sum_{j=1}^{k}(k-j+1)T\cdot Q'_j(\mathbf{x}^{\mathrm{L}}) + \sum_{\tau=1}^{k}\left(T\cdot\bar{R}'_\tau(\mathbf{x}^{\mathrm{L}}) - \epsilon kD\right) \\
&= T\cdot\left[\sum_{j=1}^{k}(k-j+1)Q'_j(\mathbf{x}^{\mathrm{L}}) + \sum_{\tau=1}^{k}\bar{R}'_\tau(\mathbf{x}^{\mathrm{L}})\right] - \epsilon k^2 D \\
&= T\cdot g'(\mathbf{x}^{\mathrm{L}}) - \epsilon k^2 D.
\end{aligned}$$

This completes the proof of the statement of the theorem. □

Using these new inequalities, the value of $\delta(\epsilon)$, which appears in Theorem 3, Corollary 1, Theorem 5 and Theorem 6, should be changed to $\delta(\epsilon) = 2\epsilon k^2 D$ accordingly. Additionally, we derive in Theorem 6 an upper bound of the ratio $V(\hat{\mathbf{x}})/V^*$ as $1+\left(\frac{\delta(\epsilon)}{k}+\epsilon D\right)/V^*$. With the new expression of $\delta(\epsilon)$, we can show that:

$$\left(\frac{\delta(\epsilon)}{k}+\epsilon D\right)/V^* \leq (2k+1)\,\epsilon D\cdot\frac{2}{(k+1)D} \leq 4\epsilon.$$

Therefore, the ratio $V(\hat{\mathbf{x}})/V^*$ is at most $1+4\epsilon = 1+\epsilon'$ for $\epsilon' = 4\epsilon$.

REFERENCES

[1] D. S. Hochbaum and X. Rao, The replenishment schedule to minimize peak storage problem: The gap between the continuous and discrete versions of the problem, *Operations Research*, 67 (2019), pp. 13451361.